The distance from the nonconstant function \( u \) in \( L^\infty(T) \) to the set \( F_{\text{const}} \) of all constant functions is estimated in terms of Hankel operators on the Hardy space \( H^2(D) \) over the unit disk \( D = \{ z \in \mathbb{C} : |z| < 1 \} \). More detailly is discussed a partial case \( \theta \in H^\infty \). More precisely, in this case we prove that \( \text{dist}(u, F_{\text{const}}) \geq \sup \| \theta \|_{\infty} \). We give a sufficient condition ensuring the equality \( \text{dist}(u, F_{\text{const}}) = \| \theta \|_{\infty} \). Moreover, we investigate the maximal numerical range and maximal Berezin set for some Toeplitz operators. Also we present some other results related with applications of Berezin symbols, in particular, we characterize normal operators in terms of reproducing kernels and Berezin symbols, and prove an inequality for the Berezin number of operator on \( H^2(D) \).