The theoretical neutron–production cross–sections produced by $^{181}$Ta($^3$He,xn)$^{184-x}$Re reactions ($x = 1$ to -7) for structural fusion material $^{181}$Ta in $^3$He–induced reactions have been performed in the incident $^3$He energy range of 14–75 MeV. Reaction cross–sections, based on theoretical pre–equilibrium nuclear reaction models, have been calculated theoretically by means of the TALYS 1.6 two component exciton, EMPIRE 3.1 exciton, ALICE/ASH geometry dependent hybrid (GDH) and ALICE/ASH hybrid models. The neutron–production cross–section results of the models have been compared with the each other and against the experimental nuclear reaction data (EXFOR). Except the $^{181}$Ta($^3$He,2n)$^{182}$Re and $^{181}$Ta($^3$He,7n)$^{177}$Re reactions, the ALICE/ASH cross–section calculations show generally agreement with the experimental values for all reactions used in this study. The ALICE/ASH–GDH model can be suggested, if the experimental data are unavailable or are improbably to be produced because of the experimental troubles.