The concept of statistical cluster point of a sequence which presents random deviations is very important in the study of optimal paths and turnpike theory. In this study, we investigate some properties of the set of all strong statistical cluster points of a sequence in a probabilistic normed (PN) space in which the norms of the vectors are represented by probability distribution functions due to randomness. In this context, we also introduce the concept of strong $\ell$-statistical convergence in a PN space, and examine some of its basic properties.