In this study, three-dimensional problem of the theory of elasticity (3DPTE) for radially inhomogeneous (INH) transversally-isotropic thin hollow spheres is investigated using the asymptotic integration method. The basic relations and equilibrium equations for radially inhomogeneous transversally-isotropic thin hollow spheres are formed and inhomogeneous solutions (INHSs) and homogeneous solutions (HSs) are constructed. The built solutions completely reveal the qualitative structure of a three-dimensional stress-strain state of radially inhomogeneous transversely-isotropic spheres of small thickness and serve as an effective apparatus for solving boundary value problems, the basis for evaluating existing applied theories and for creating new, more refined applied theories. Asymptotic formulas are obtained that allow the calculation of the three-dimensional stress-strain state of spheres. New solution groups (boundary layer solutions) have been found that are absent in applied theories. The behavior of homogeneous solutions in the inner parts of the sphere and in the vicinity of conical sections has been studied, when the thin-walled parameter of the sphere tends to zero. The nature of the constructed homogeneous solutions is clarified. On the basis of the theoretical analysis, three types of the stress-strain state (SSS) in the radially inhomogeneous transversally-isotropic hollow spheres (RINHTIHSs) are considered: a penetrating stress state, a simple edge effect, and a boundary layer. Finally numerical calculations are made and the influences of inhomogeneity on the stress distributions are investigated.